International Journal of Emerging Engineering Research and Technology
Volume 3, Issue 6, June 2015, PP 54-62
ISSN 2349-4395 (Print) & ISSN 2349-4409 (Online)

Modeling and Analysis of Tri-Copter (VTOL) Aircraft

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ABSTRACT

The purpose of this research is to provide the concepts, dynamics and control strategies of tri-rotor unmanned aerial vehicles. A device which is used or intended for flight in the air that has no on-board pilot is called Unmanned aerial vehicles (UAVs). UAVs includes many devices such as tri-copter, qua-copter. Among them, tri-copter (VTOL) aircraft is proposed in this paper. A tri-rotor UAV has three rotor axes that are equidistant from its center of gravity. In this research, a tri-copter consists of body frame of reference and earth frame of reference. Therefore, mathematical modeling and analysis of tri-copter (VTOL) is done. The kinematic and dynamic analysis for a tri-copter mini-rotorcraft is to compute new results for tri-copter. According to the parametric equation, the orientation and control of tri-copter can be shown. Thus, the relation or dependence of position on the angular speed of motor/s is also shown by plotting three dimensional positions versus time graph. The results with different cases can solve by MATLAB. Then, a control strategy is proposed for each type of tri-rotor, and nonlinear simulations of the altitude, Euler angle, and angular velocity responses are conducted by using a classical proportional-integral-derivative controller.

Keywords: Tri-copter, Mathematical Model, Control strategy, MATLAB, Moment of inertia.

INTRODUCTION

To encourage missions in dangerous situations, flying stages that are little, spry and can take of vertically are of investment. A stage that satisfies these necessities is a UAV (Unmanned Aerial Vehicle) as a multi copter joined with a great control framework. A multi copter is a rotor craft that has more than two rotors, in light of the fact that a rotorcraft with two rotors is called helicopter (bicopter). Multi copters have altered cutting edges with a pitch that are not conceivable to control, as it accomplishes for a helicopter (through the swash plate), to control the bearing of the rotor push. Rather, the rates of the rotors are shifted to accomplish movement control of a multi copter. For a tricopter, there is additionally a servo joined that can tilt one of the engines and by that accomplish a change in movement. Multi rotor air ships are regularly utilized within model and radio controlled activities due to the straightforward development and control. Because of the amount of rotors, the extent of them doesn't have to be substantial in examination with a helicopter that just has one rotor to actuate enough drive to lift it up. UAVs are frequently utilized within spots where it is troublesome for a man to work in, for example, perilous situations, soak landscape and so forth. It is likewise a suitable and a modest apparatus for observation. A Polaroid mounted on a multi copter is an exceptionally adaptable approach to study a region. UAVs can without much of a stretch be intended for a particular mission and all things considered it is an exceptionally adaptable apparatus. This theory is about displaying and dissection of a tricopter. It is an airplane with three rotors and a tail servo. This has the focal point of a helicopter with brisk yaw developments.

Rigid-Body Equation of Motion

Considering the size of tri-rotor UAVs relative to the surroundings, tri-rotor UAVs are assumed to be rigid objects. The 6-degree of freedom (DOF) nonlinear equations of motion that are developed for conventional UAVs are also used for tri-rotor UAVs. The tri-rotor UAV is free to rotate and translate in three-dimensional space, and the rigid body dynamics are derived by Newton's laws). For tri-rotor UAVs, the 6-DOF rigid body equations of motion are expressed as the differential equations describing the translational motion, rotational motion, and kinematics as given below.

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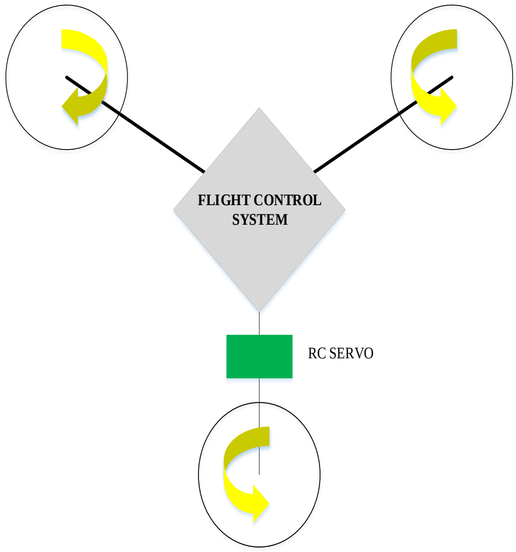

Figure1. *The Flight Principle of the Tri-Rotor*

Force equations

[1]$\dot{u}$=rv-qw-g sin θ+F_X/m

[2]$\dot{v}$=-ru-pw-g sin ∅ cos θ +F_y/m

[3]$\dot{w}$=qu-pv-g cos ∅ cos θ +F_z/m

Moment equations

[1] $\dot{p}=(I_{yy}-I_{zz})qr/I_{xx}+L/I_{xx}$

[2] $\dot{q}=(I_{zz}-I_{xx})pr/I_{yy}+M/I_{yy}$

[3] $\dot{r}=(I_{xx}-I_{yy})pq/I_{zz}+N/I_{zz}$

Kinematic equations

[1]$\dot{\emptyset}$=p+q sin ∅ tan θ +r cos ∅ tan θ

[2]$\dot{\theta}$=q cos ∅ -r sin ∅

[3]$\dot{\Psi}$=(q sin ∅ +r cos ∅) sec θ

In the above, F_x,F_y,$F_z$are the external forces, and (L,M, N) are the external moments acting on the center of gravity with respect to the body-fixed frame. (u, v, w) are the translational velocities, (p, q, r) are the rotational velocities, and (∅, θ, ψ) are the rotational angles. I_{xx},I_{yy} and I_{zz} are the rotational inertias of the tri-rotor UAV.

MATHEMATICAL MODEL

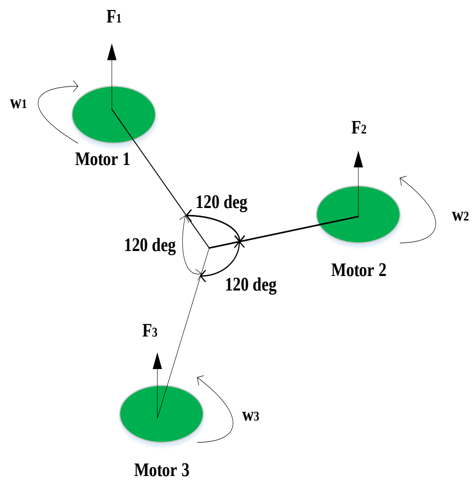

Figure2. *Three Rotor Rotorcraft Scheme*

The above figure is a diagram of the dynamic equation of the tricopter in which the lower right side is the diagram of the dynamic equation of yaw control. That is because in yaw control, RC servomotor drives the tail axis to change the declination angle of the tail axis.

The tri-rotor's motion control can be decomposed into altitude, roll, pitch, and yaw control. The control strategies of tri-rotors are shown in Fig. 3. Fig. 3(a) shows the altitude control and that increasing the speed of each rotor will increase the altitude, and vice versa. Fig. 3(b) shows the roll control; the approach towards roll control is that given the same rotor-1 speed, varying the rotor speeds of the two front rotors will generate roll control. Figure 3(c) shows the pitch control; given the same angular velocities for the front two rotors, varying the rotor speed of rotor 1 will generate pitch control. Regarding the yaw control, by using the natural yawing moment from the reaction torque and also from the tilt angle, yaw control can be successfully generated. The tilt angle is very useful when encountering a sudden danger of collision because by tilting the rotor, sudden turning control would be possible.

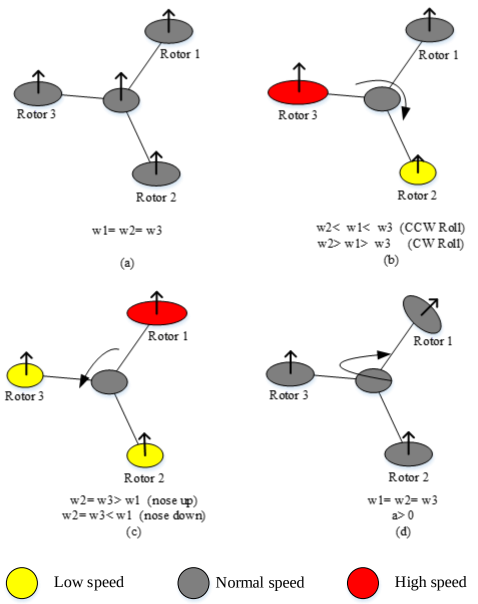

Figure3. *Control strategies for tri-rotors, (a) altitude, (b) roll, (c) pitch, (d) yaw*

Moment of Inertia of Tricopter

Figure 4 is the diagram of the moment of inertia of the tri-rotor. When calculating the inertia torque of the tri-rotor, we assume the fuselage is rectangular shape, the three motors are cylindrical in shape; furthermore, the inertias of the round rods of the axes are neglected.

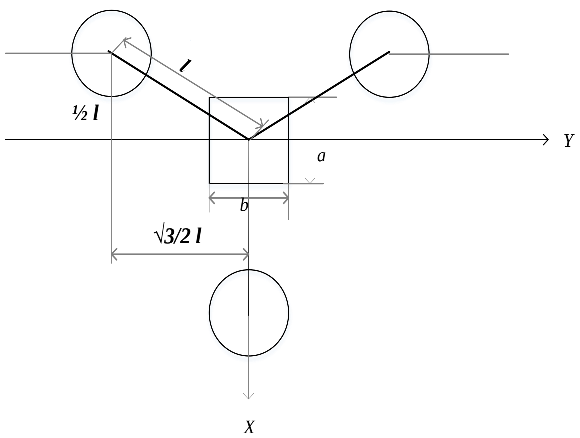

Figure4. *Moment of inertia of each axis*

The total moment of inertia about *x*-axis$=\frac{3}{2}ml^2+\frac{1}{12}mob^2+\frac{1}{12}m(3r^2+h^2)$

Moment of inertia about axis y: The total moment of inertia about y-axis: $I_{yy}=\frac{3}{2}ml^2+\frac{1}{12}moa^2$

Moment of inertia about axis z: The total moment of inertia about z-axis: $I_{zz}=\frac{1}{12}mo(a^2+b^2)+3ml^2$

Coordinate System

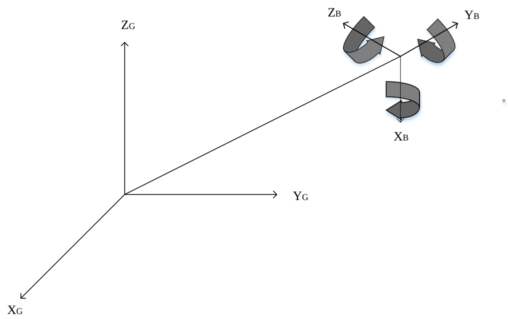

Figure 5. *Coordinate of the Tri Copter*

There are 3 different co-ordinate systems of interest that needs to be defined. The local, which is seen as the body fixed co-ordinates of the tricopter and denoted by B and global, which is co-ordinate system of earth and denoted by G.The tricopter has three arms formed as Y and a rotor is placed at the end of each arm. The coordinate system for tricopter will be defined as in fig 5.

The X_B axis is defined in the direction straight ahead seen from the view of the tricopter and Y_B axis to the right. This axis is defined straight down from the center of mass of the tricopter. The Y_B axis can be seen as the desired forward direction during a flight. The relation between tricopter's(Body) co-ordinate system and co-ordinate system of earth can be described in a mathematical way with the rotation matrix $Q_{xyz}^{\emptyset\theta\Psi}$. The rotation is represented with euler angles which is represented with ($\emptyset$, θ, Ψ) and is the angle around X_B, Y_B and Z_B axes respectively. The rotation applied to each of the base vectors and the total rotation is done by first rotating the Z_B axis with an angle Ψ, then done by first rotating the Y_B axis with an angle θ and at last the XB axis with an angle $\emptyset$.

$$Q_{xyz}^{\emptyset\theta\Psi}=Q_{\emptyset}^{x}Q_{\theta}^{y}Q_{\Psi}^{z}=\begin{pmatrix}1 & 0 & 0\\ 0 & c\emptyset & s\emptyset\\ 0 & -s\emptyset & c\emptyset\end{pmatrix}\begin{pmatrix}c\theta & 0 & -s\theta\\ 0 & 1 & 0\\ s\theta & 0 & c\theta\end{pmatrix}\begin{pmatrix}c\Psi & s\Psi & 0\\ -s\Psi & c\Psi & 0\\ 0 & 0 & 1\end{pmatrix}=\begin{pmatrix}c\theta c\Psi & c\theta s\Psi & -s\theta\\ s\emptyset s\theta c\Psi-c\emptyset s\Psi & s\Psi s\theta s\emptyset-c\emptyset c\Psi & s\emptyset c\theta\\ c\emptyset c\theta c\Psi+s\theta c\Psi & s\Psi s\theta c\emptyset-c\emptyset c\Psi & c\emptyset c\theta\end{pmatrix}$$

Physical motion model

To be able to express the dynamical model of the aircraft, the translational and rotational equations have to be derived.

Translation

Mass multiplied by with the time derivative of the translational speed vector is equal to the directional force vector;

$$m\dot{V}_G=\sum f$$

The gravitational force is separated from the externals because it is acting on the mass centreand is therefore not inducting any torque on the centre of mass. The force equations are

$$m\dot{V}_G=\sum f=f_G+f_g(\text{gravity})=Q_{xyz}^{\emptyset\theta\Psi}(f_B)_{ext}+[0\ 0\text{-}mg]$$

Rotational

We derive the rotational equation of motion from Euler's equation for rigid body dynamics by considering the tricopter as a rigid body in the body frame

$$\tau = I\dot{w}$$

$$\dot{w}=\begin{pmatrix}\dot{w}_x\\ \dot{w}_y\\ \dot{w}_z\end{pmatrix}$$

$$I=\begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{pmatrix}$$

$$\tau=\begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{pmatrix}\begin{pmatrix} \dot{w}_x \\ \dot{w}_y \\ \dot{w}_z \end{pmatrix}$$

FABRICATION OF THE TRICOPTER

Electronic Components

ESC

An electronic speed controller or ESC is an electronic circuit with the reason to fluctuate an electric engine's speed its bearing and perhaps at the same time to act an element brake. ESCs are frequently utilized on electrically fueled radio controlled models, with the assortment regularly utilized for brushless engines basically giving an electronically produced three stage electric power low voltage wellspring of vitality for the engine.

Escs intended for radio-control planes/copters generally hold a couple of wellbeing characteristics. In the event that the force originating from the battery is lacking to keep running the electric engine will decrease or cut-off force to the engine while permitting proceeded utilization of ailerons, rudder and lift capacity. This permits the pilot to hold control of the plane to float or fly on low power to wellbeing.

Specification:

Continuous Current Rating= 18 Amp

Burst Current Rating= 22 Amp

BEC Mode= Linear

BEC= 5V/ 2 Amp

LiPo Cells = 2-4 Cells

Weight= 19 grams

Size= 24x45x11mm

Brushless DC motors

The DC-engine is an actuator which changes over electrical vitality into mechanical vitality (and the other way around). It is made out of two intuitive electromagnetic circuits. The first (called rotor) is allowed to turn around the second one (called stator) which is settled. In the rotor, a few gatherings of copper windings are associated in arrangement and are remotely open because of a gadget called commutator. In the stator, two or more lasting magnets force an attractive field which influences the rotor. By applying a DC-current stream into the windings, the rotor turns in view of the energy produced by the electrical and attractive association.

A dc motor can be extensively characterized into two recognized sorts of engines specifically:

- Brushed dc motor
- Brushless dc motor

For our project we will be using brushless dc motor.

Specification:

Kv (rpm/v) = 1000

Weight= 56 grams

Max Current = 17 Amps

Max Voltage= 11V

Motor Length= 32mm

Motor Diameter = 28mm

Total Length= 46mm

Flight Controller Board

The KK multicopter controller is a sort of flight control framework, which might be connected to a multirotor air craft with distinctive axes, including single-hub, double-pivot, tri-hub, quadhub, hex-hub, eight-hub, and additionally the flying machine with fixed wings. The KK multicopter flight controller has an Atmega microchip and tri-pivot gyro that can discover the rakish speed of move, pitch, and yaw headings, and 8-channel PWM sign yield. It can control eight engines or RC servos at most so that the air ship can fly steadily.

Basic Functions Of KK Multicopter Flight Controller

The tri-hub gyro-settled framework has contra-turning gyro chips, with the capacities of electronic alteration and adjustment of the quickening agent pedal and locking rotection. The equipment construction modeling of the flight controller connected on the tricopter in which the remote control recipient is utilized to accept the remote control indicator sent from the radio controller. Likewise, the MEMS gyro could be utilized to catch the rakish speed of the headings of move, pitch, and yaw for the three rotors. The mimicked voltage yield by the gyro could be specifically perused by the single chip at mega 168-20au on the flight controller, taking into account which the hub declination plot of the three rotors are figured. The single chip on the flight controller can control the velocity contrast between the three engines, and the declination point of the RC servos, with a specific end goal to keep up the offset motion of the three rotors.

Li-Po Battery

Lithium Polymer (Lipo) cells are one of the most up to date and most revolutionary battery cells accessible. Lipo cells keep up a more reliable voltage over the release bend when contrasted with Nicd or Nimh cells. The higher ostensible voltage of a solitary Lipo cell (3.7v and 1.2v for a normally Nicd or Nimh cell), making it conceivable to have a proportionate or considerably higher aggregate 23 ostensible voltage in a much littler bundle. Lipo cells ordinarily offer high limit for their weight, conveying upwards of double the limit for ½ the weight of practically identical Nimh cells. In conclusion, a Lipo cell battery needs to be deliberately observed throughout charging since cheating and the charging of a physically harmed or released cell could be a potential flame peril and perhaps even deadly. Li-poly batteries are picking up support in the realm of radio-controlled airplane due to its preferences of both easier weight and significantly expanded run times. A urging playing point of Li-poly cells is that makers can shape the battery practically any way they kindly which could be vital in their use in micro airplanes and rotors.

Specification:

Capacity = 3000 mAh

Voltage= 3 Cell 11.1 V

Discharge= 20C Constant &30C Burst

Weight = 306 grams

Dimensions = 146x51x22mm

Max Charge Rate= 2C

VELOCITY MODEL

Let $\sum F$ be the total force acting on the tricopter. This $\sum F$is due to resultant of gravity force and thrust force i.e.

$$\sum F = F_{thrust} + F_{gravity}$$

F_{thrust} is provided by the velocities of the motors mounted at the extremes of the three ends/hands of the tricopter. Experimental results have shown that F_{thrust} is directly proportional to w_i^2

$$F_{thrust} = k\left(w_1^2 + w_2^2 + w_3^2\right)$$

We need to find out a relation between angular velocity of the tricopter and the linear velocity in order to predict the position of the tricopter with time within an interval. The relation is given by Newton-Euler equation: $\begin{pmatrix} F \\ \tau \\ 0 \end{pmatrix} = \begin{pmatrix} m & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a \\ \alpha \\ 0 \end{pmatrix} + \begin{pmatrix} w \times mv \\ w \times Iw \\ 0 \end{pmatrix}$

Where

F=total force

m=mass

τ = total torque

a=linear acceleration

α=angular acceleration

I=moment of inertia

v=linear velocity

w=angular velocity

$$F=ma+mvw$$

$$\tau=I\alpha+Iw^2$$

From above eq, we can conclude that

$$V=\frac{F-ma}{mw}$$

Assuming acceleration to be zero

$$V=\frac{F}{mw}$$

Where

$F=K(w_1^2+w_2^2+w_3^2)$, K= constant

$$v=\frac{K'\left(w_1^2+w_2^2+w_3^2\right)}{w}$$

$$v^2=K''\left(w_1^2+w_2^2+w_3^2\right)$$

Since w=v/r

Now acceleration of the system

$ma=F-mvw$

$a=\frac{F-mvw}{m}$

$=\frac{K\left(w_1^2+w_2^2+w_3^2\right)-mv(w_1+w_2+w_3)}{m}$

$=K'(w_1^2+w_2^2+w_3^2)-m(v_x+v_y+v_z)(w_1+w_2+w_3)$

Where K'=K/m=const

$v=\sqrt{K''(w_1^2+w_2^2+w_3^2)}$

$\vec{v}=v(\cos a\vec{i}+\cos b\vec{j}+\cos c\vec{k})$

Taking a,b,c as const=45 degree, we get

$\vec{V}_x=\vec{i}\sqrt{K(w_1^2+w_2^2+w_3^2)}$

$\vec{V}_y=\vec{j}\sqrt{K(w_1^2+w_2^2+w_3^2)}$

$\vec{V}_z=\vec{k}\sqrt{K(w_1^2+w_2^2+w_3^2)}$

Simulation in Mat lab for Position Determination

The relation or dependence of position on the angular speed of motor/s is shown by plotting three dimensional positions versus time graph in MATLAB. The figure 6 shows that the screen shot of the plot of position versus time. When acceleration is zero, angular velocity is constant. In order to determine the position of the tri-copter with time within an interval, it is need to solve a calculation between angular velocity of the tri-copter and the linear velocity. When acceleration is zero, angular velocity is constant. According to simulation result, the angular velocity is 50 as acceleration is zero. Although the angular velocity is also 70, acceleration .Then, acceleration can be zero when the angular velocity is 100. The value of angular velocity occur 150 when acceleration is 0. Finally, the angular velocity reach's as 200. These results display 3D-position with the help of MATLAB.

Case1. *Angular velocity is constant and acceleration is zero*

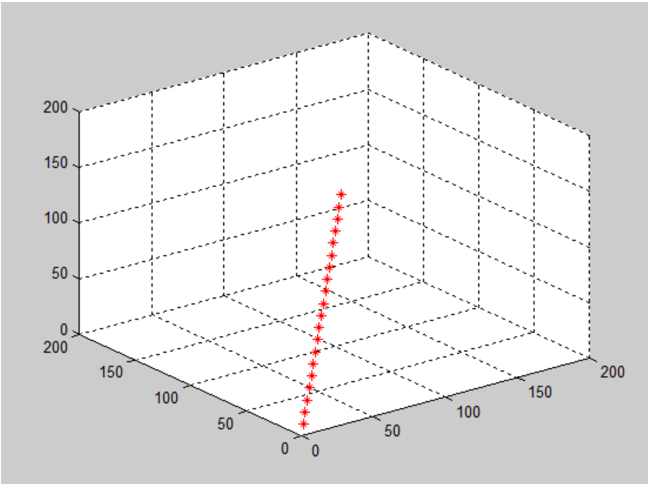

Figure6. *Plot of position versus time when angular velocity is constant and acceleration is zero*

The screen shot describes the plot of position versus time when motors are rotating with a constant angular acceleration as shown in figure 7.When angular acceleration is 50, motor can rotate with position various time.

Case2. *Motors are rotating with a constant angular acceleration*

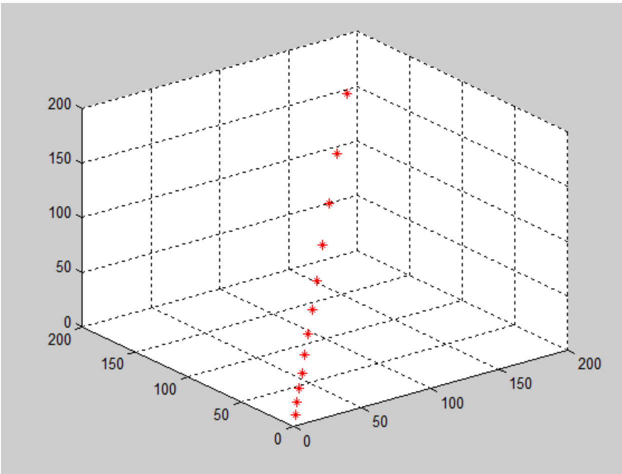

Figure7. *Plot of position versus time when motors are rotating with a constant angular acceleration*

CONCLUSION

In this paper, we carried out to bring out the Mathematical Model of the tricopter. MATLAB simulation have been predicted the position of the tricopter during flight. Two separate graphs were plotted in MATLAB with varying conditions. In addition to this, specification of each parts were found out and noted down in this report. Thus, the importance of a small-sized vertical take-off and landing (VTOL) UAV for various missions was mentioned, and tri-rotor UAVs were introduced as one of the examples. The tri-rotor UAV can solve the yawing moment problem by tilting the tail servo motor.

ACKNOWLEDGMENT

The author would like to thank to Dr. Hla Myo Tun, Associate Professor and Head of the Department of Electronic Engineering, Mandalay Technological University for his help and for his guidance, support and encouragement. In particular, the author would like to thank U Khun Karry, my father; Daw Nang Myint Ohm, my mother; Ma Nang April Ju, my elder sister; and Mg Khun Joseph, my youngest brother; for their complete support.

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